THE FALL OF POTENTIAL IN THE INITIAL STAGES OF ELECTRICAL DISCHARGES

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Abstract

The fall of potential in the initial stages of condensed discharges through air, nitrogen, hydrogen and CO_2 has been studied as a function of the pressure. The experimental method consisted in measuring the magnitude and shape of the potential wave impressed upon two parallel, copper wires attached to the electrodes of the discharge. In every case the rate of potential fall was increased with increase of pressure. Over the pressure range studied (50 to 140 cm of mercury) the decrease of potential was slowest in CO_2 of the gases quantitatively studied. However, rough measurements showed the rate of fall to be very much slower in helium than in CO_2. In hydrogen a small overvoltage produced an increase in the rate of potential fall at all pressures. The results are shown to be in good agreement with Toepler's discharge law. A method is described by means of which it is possible to apply 5×10^5 volts/cm across a spark gap in air, hydrogen or nitrogen at atmospheric pressure at least 10^{-6} sec. without electrical breakdown. When such discharges once are initiated the potential falls much faster than for a static breakdown.

THE various types of electrical discharges through gases have been extensively studied in the past and remarkable progress has been made in explaining the numerous phenomena in terms of well recognized elementary processes.[1,2] However, practically all of the great mass of experimental data has been obtained in connection with discharges that had already reached a steady state and, therefore, gives little direct information about the initial stages of the discharge. This state of affairs results from the fact that in general the electrical breakdown takes place so quickly that proper apparatus with sufficient time resolving power has only within the last few years been devised. The factors, in the initial stages of the discharge, which remain to be experimentally investigated must obviously include the potential gradient in the discharge, the current density in the gas, the size and shape of the discharge together with the spectra emitted, all studied as functions of the time. Some progress has already been made in the study of the last three factors by means of Kerr cells[3,4,5] and rotating mirrors but a satisfactory direct way of measuring the actual electric field and current density throughout the discharge in such short times, does not exist, However, the cathode-ray oscil-

[1] J. J. Thomson, Conduction of Electricity through Gases, 2nd Ed. 1906; J. J. Thomson and G. P. Thomson, Conduction of Electrciity through Gases, 3rd Ed. 1928.

[2] Compton and Langmuir, Rev. Mod. Phys. **2,** 123, (1930); **3,** 190 (1931).

[3] Beams, Phys. Rev. **28,** 475 (1926); **35,** 24 (1930).

[4] Lawrence and Dunnington, Phys. Rev. **35,** 396 (1930).

[5] L. V. Hamos, Ann. d. Physik **7,** 875 (1930).

lograph[6,7] has been improved until it will give an approximation to the rate of change of potential across the discharge and, with a much less degree of precision, the rate of change of total current through the discharge. These data, though not yielding directly the electric field and current density, give valuable information from which it is possible to construct and roughly to test theories of electrical breakdown. The experimental arrangements in such cathode-ray experiments require a large amount of apparatus coupled with special technique and skill in operation. Further the oscillograph possesses inherently a certain finite capacity in its deflecting plates as well as inductance in the leads to them so that the results are often difficult to interpret. Several other methods[8,9,10] of getting the rate of fall of potential across the discharge have also been used with some success.

We have developed two comparatively simple methods of investigating the time required for the potential to fall in discharges at various pressures of the gas. The two methods supplement each other, the first giving the total length of time required for the potential across the discharge to fall to a steady value and the second giving the potential across the gap as a function of the

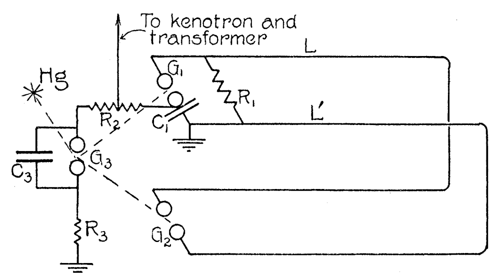

Fig. 1. Schematic diagram of apparatus used in the first experimental method.

time. Both methods are based upon the fact that a potential wave is impressed upon the lead wires during the breakdown. From a measure of the maximum potential attained between certain parts of the circuit the magnitude and shape of the potential wave could be calculated from circuital theory and thus the rate of fall of potential across the discharge secured. Somewhat similar methods have been used previously by Toepler[11] and Binder[8] but the present methods differ from theirs in some important essentials.

The first experimental arrangement is illustrated schematically in Fig. 1. The condenser C_1 of about 0.003 mf capacity, slowly charged by a transformer and kenotron, is the source of energy for the discharge to be studied at G_1. R_1 is a high resistance (about 10^5 ohms) which serves to prevent the line L from floating. The leads LL' are two straight parallel copper wires (No. 12,

[6] Dufur, Onde Elec. **2,** 19 (1923).

[7] Rogowski and Flegler, Arch. f. Elektrot. **15,** 207 (1925); Rogowski, Flegler and Tamm, Arch. f. Elektrot. **18,** 513 (1927); Beyerle, Arch. f. Elektrot. **25,** 267 (1931).

[8] See Binder, Wanderwellenvorgänge, Berlin, 1928.

[9] Lawrence and Beams, Phys. Rev. **32,** 483 (1929).

[10] Dunnington, Bul. Amer. Phys. Soc. **A2,** April 16, 1931.

[11] Toepler, Archiv. f. Elektrot. **14,** 305 (1925); **17,** 61 (1926); **18,** 549 (1927); **21,** 433 (1929).

25 cm apart). The gap G_2 (low capacity) is used as an indicator to measure the maximum potential across the ends of the wires. The condenser C_3 and gap G_3 are used in order to irradiate G_2 from a strong source of ultraviolet light, namely the discharge which occurs at G_3. For this arrangement the time lag of the indicating gap has been shown in this laboratory to be extremely short. G_1 and G_3 are set at the proper spacing in order that, when charged from the same system, they reach a breakdown field strength at the same time. Due to the irradiation from the mercury arc lamp, Hg, G_3 discharges first. G_1 (carefully shielded from Hg) is then intensely irradiated by G_3 and immediately discharges. As the breakdown in G_1 progresses, a potential wave, the shape of which is determined by the discharge characteristics of the gap, is impressed on leads LL'. (The distortion of this wave as it progresses along the wires can be computed from the known constants of the wires.) This wave travels along LL' at approximately the velocity of light until it reaches G_2 where its potential is progressively doubled at reflection. The maximum

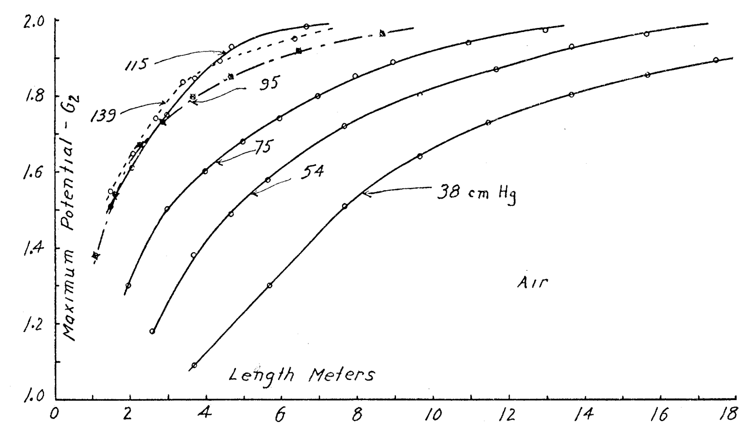

Fig. 2. Results obtained, with the first method, for air at various pressures. The maximum potential at G_2 plotted against the length of the leads LL'.

sparking width of G_2 (which is intensely irradiated from the bright spark G_3) serves as a measure of the maximum potential attained across the ends of the leads.

The curves, Fig. 2, show the values of the maximum potential at G_2 plotted against the length of LL' for various air pressures in G_1. The ordinates are reduced to a scale in which the breakdown potential of G_1 is taken as unity. When this scale is used it is found that the curves for different widths of G_1 (between 2 and 6 mm) are identical within experimental error if the air pressure surrounding G_1 is held constant.

The decrease in potential at G_2 as the leads are shortened may be qualitatively explained as follows. The reduction in maximum potential at G_2 is due to interference from the front of the wave which, after reflection there, returns to G_1, suffers a reverse reflection, and again reaches G_2 before the wave there has attained its maximum value. In order for this reversed wave to reduce the double value of the potential at the end, it is necessary that the time length of the wave be greater than the time equivalent of twice the wire length. A careful analysis of the circuit shows that although it is possible to determine with precision the total length of the potential wave by this method, the actual potential-time curve cannot be obtained unless we know the law connecting the change of resistance in G_1 with time. The necessity for this assumption arises from the fact that the reflection coefficient at G_1 must depend upon its resistance which, in turn, is a function of the time. It will be noted from the curves that the total time required for the potential to fall to a steady value decreases with increasing pressure over the range studied. We will return to a discussion of Fig. 2 after first describing the second method.

The second method is shown in Fig. 3. The discharge at G_1 impresses an electric wave on the wires LL' in the same manner as before. It travels to the end which is closed by a heavy conductor P, suffers a reverse reflection, returns to G_1 where it is reversed again at reflection and the process repeats itself until the waves are damped out. The leads LL' were always greater

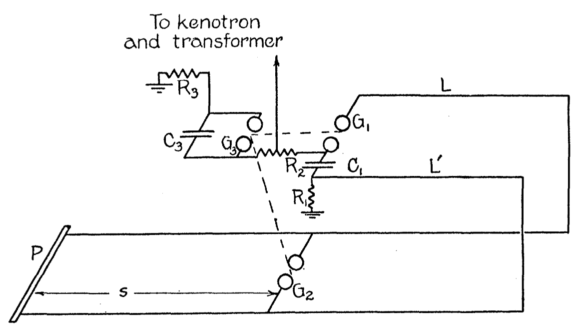

Fig. 3. Schematic diagram of apparatus used in the second method.

than half the length of the potential wave so that the resistance of G_1 could reach a low value before the first reflection occurred, thus reducing the damping. An indicating gap G_2 is placed a distance s from the closed end. This gap is irradiated from an independent discharge G_3 in the same manner as previously described. Since the time lag of G_2 is small and the damping of the oscillations not large, the sparking width of G_2 gives a direct measure of the maximum fall of potential during a time interval $2s/c$ where s is the distance of G_2 from the closed ends and c is the velocity of the electrical impulse along the wires. It may be worth while to mention our procedure in obtaining the maximum sparking potential of G_2. The transformer was adjusted so that G_1 would spark about every two seconds. The width of G_2 was then increased until sparking across it did not quite cease (one discharge across G_2 to twenty five across G_1). This point was taken as a measure of our potential. It may be significant to note that the distance necessary to widen G_2 from a condition where every spark at G_1 produced a spark at G_2 to the point where only one discharge occurred at G_2 to twenty five at G_1 was least when G_1 was in nitrogen and greatest when it was in CO_2.

Fig. 4 shows the maximum potential across G_2 plotted as a function of $2s/c$ for various pressures of air, hydrogen, CO_2 and nitrogen. It will be observed that the rate of fall of potential increases with pressure, i.e., as the pressure in G_1 is increased the electrical wave impressed upon the wire becomes steeper.

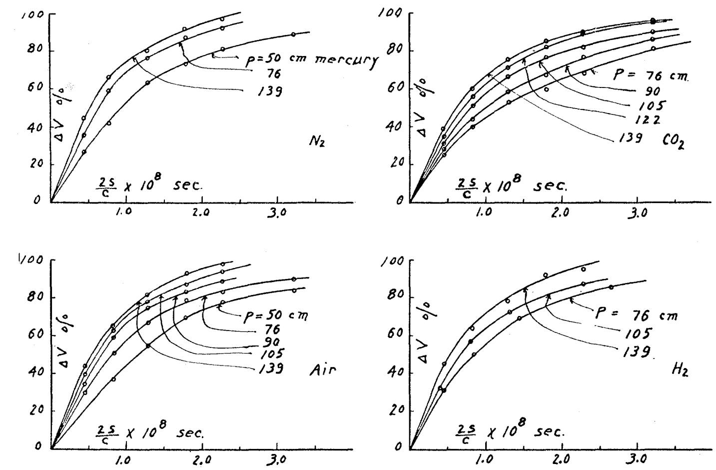

Fig. 4. Results obtained with the second method for air, CO_2, H_2 and N_2 at various pressures. The maximum potential across G_2, ΔV, plotted against the time interval $2s/c$.

Further it will be noted that for small values of $2s/c$ the ratio of the voltage across G_2 to $2s/c$ remains almost constant indicating that the potential when changing most rapidly is nearly a linear function of the time.

In formulating an analysis of the curves in Fig. 4 we were guided by the work of Toepler.[11] He has established a law for his *Gleitfunken* (surface discharge) from which it is possible to find the resistance R_t of the discharge at any time t after the beginning from the relation

$$R_t = \frac{K \cdot \delta}{\int_0^t idt} \tag{1}$$

where K is Toepler's spark constant δ the discharge length and $\int_0^t idt$ the charge which has passed during the time t. This law has been extrapolated to the case of the spark in ordinary air, and Binder[8] has obtained values for K at atmospheric pressure. Using this law of Toepler's Schilling[12] has worked out a solution for the circuit in which we are interested and shows that

$$t = \frac{K \cdot \delta}{E} \left\{ \frac{V}{E - V} - \frac{1}{2} + \log \frac{2V}{E - V} \right\} = \frac{K \cdot \delta}{E} f(V) \tag{2}$$

in which t is the time and E the breakdown potential of the gap. This equation was derived on the assumption that the capacity C_1 multiplied by the

[12] Schilling, Archiv. f. Elektrot. **25**, 2, 97 (1931).

surge impedance of the leads was large in comparison to any time considered. The constants are evaluated so that t is zero when dv/dt is a maximum (coincident with the point $V=E/3$). In order to simplify the calculations the scale of E and V have been arranged so that E is 100 and accordingly the values of V are in percent. A graph of the relation is shown in Fig. 5. The values of $K\delta/E$ are obtained from Eq. (2) by first determining the values of V_1 and V_2

TABLE I. *Values of ΔV and $K\delta/E$ for air, CO_2, and N_2 at various pressures.*

Pressure cm mercury	Time interval $2s/c\times10^8$	Air ΔV percent	Air $K\delta/E$ $\times10^9$	CO_2 ΔV percent	CO_2 $K\delta/E$ $\times10^9$	N_2 ΔV percent	N_2 $K\delta/E$ $\times10^9$
		($E=14.2$ kv)		($E=$—)		($E=15.1$ kv)	
50	0.45					27	2.3
	0.83	37	3.0			42	2.6
	1.29	55	2.8			63	2.2
	1.79	70	2.6			73	2.3
	2.28	78	2.5			81	2.2
	3.22	84	2.7			89	2.0
		($E=17.7$ kv)		($E=15.2$ kv)		($E=14.5$ kv)	
76	0.45	30	2.1	25	2.6	36	1.7
	0.83	51	2.0	40	2.7	59	1.6
	1.29	67	2.0	53	3.0	76	1.5
	1.79	79	1.9	59	3.5	87	1.3
	2.28	83	2.0	68	3.5	92	1.1
	3.22	90	1.8	81	3.1		
		($E=16.8$ kv)		($E=13.5$ kv)		($E=16.8$ kv)	
90	0.45	34	1.7	28	2.3	40	1.5
	0.83	59	1.6	44	2.4	60	1.6
	1.29	75	1.6	58	2.6	75	1.6
	1.79	83	1.6	67	2.8	86	1.4
	2.28	89	1.5	77	2.6	93	1.0
	3.22			86	2.2		
		($E=14.8$ kv)		($E=15.7$ kv)		($E=15.4$ kv)	
105	0.45	40	1.5	31	2.0	43	1.4
	0.83	63	1.5	51	2.0	65	1.4
	1.29	78	1.4	66	2.1	81	1.3
	1.79	87	1.3	76	2.1	92	0.9
	2.28	94	0.9	82	2.1	96	0.7
	3.22			90	1.9		
		($E=16.5$ kv)		($E=17.1$ kv)		($E=14.8$ kv)	
139	0.45	44	1.3	39	1.5	45	1.3
	0.83	65	1.4	60	1.6	66	1.3
	1.29	82	1.2	75	1.6	80	1.3
	1.79	93	0.8	85	1.5	92	0.9
	2.28	98	0.4	90	1.3	97	0.5
	3.22			96	0.9		

where $V_2-V_1\equiv\Delta V$ represents the potential measured by G_2 for each value of $2s/c$. It is easy to see that this is possible because if $(V_2-V_1)/\Delta t$ is a maximum, where $\Delta t=2s/c$, then the slopes of the curve, Fig. 5, are equal at V_1 and V_2. The values in Table I are the results of such computation for air, CO_2 and nitrogen. Those for hydrogen are in Table II. It will be observed from the constancy of $K\delta/E$ for each pressure that the data are in fair agreement with Toepler's law, except for the larger values of ΔV. However, it should be noted that in the range where the law fails to fit, a small change in V produces a large change in $f(V)$ and the errors of observation are greatly multiplied.

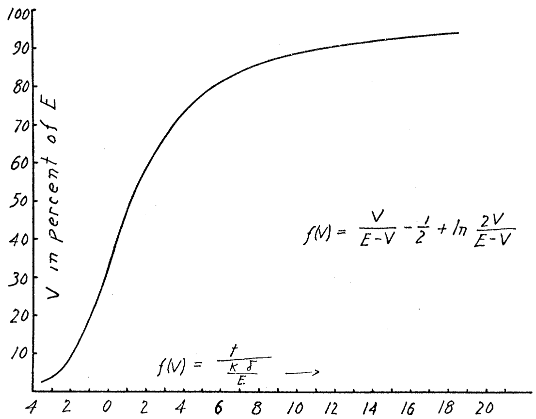

Fig. 5. Showing the variation of V with t according to Toepler's law for the case of a large condenser discharging into a long line.

From the mean values of $K\delta/E$ the variation of V with time for any pressure can be found at once by simply multiplying the scale of the absicissas ($f(V)$) in Fig. 5 by the proper value of $K\delta/E$. Fig. 6 shows such curves in-

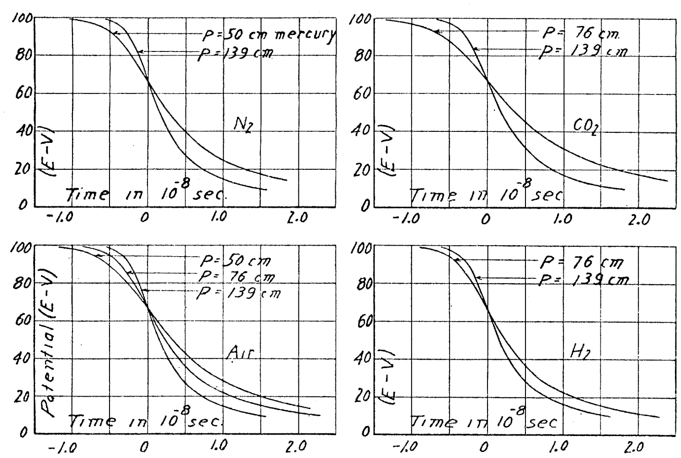

Fig. 6. The potential across the discharge, $(E-V)$, plotted against the time for air, CO_2, H_2, and N_2 at various pressures. These curves were obtained by substituting the mean values of $K\delta/E$ from Tables I and II into Eq. (2).

verted $(E-V)$ for air, hydrogen, nitrogen and CO_2. In helium qualitative results show that the fall of potential is much slower than in air.

It will be observed from Table II that in hydrogen where the potential across the discharge is allowed to rise above the sparking voltage, that $K\delta/E$

and hence K are decreased. (These measurements could easily be obtained experimentally by the proper adjustment of the gaps G_1 and G_3.) This means that the rate of fall of potential is increased by increasing the voltage above the sparking potential before the discharge is initiated. The differences introduced are quite large and should indicate that where the potential across a gap is impressed rapidly without ultraviolet light or some agent to generate electrons in the gap to prevent time lag, the rate of fall of potential may be considerably faster. This possibly accounts in part for the range of variation

TABLE II. *Values of ΔV and $K\delta/E$ for H_2 for normal breakdown and for slight overvoltages.*

Pressure cm mercury	Time inteval $2s/c\times 10^8$	H_2 ΔV percent	H_2 $K\delta/E\times 10^9$	H_2 overvolted ΔV percent	H_2 overvolted $K\delta/E\times 10^9$
		($E=12.3$ kv)		($E=12.6$ kv)	
76	0.46	31	2.1	39	1.6
	0.85	50	2.1	58	1.7
	1.44	69	2.1	80	1.5
	2.65	85	2.1	94	1.0
		($E=14.8$ kv)		($E=15.2$ kv)	
90	0.40	31	1.8	37	1.5
	0.74	50	1.8	57	1.5
	1.31	68	2.0	75	1.6
	2.02	81	2.0	89	1.4
	2.65	87	1.8	92	1.3
		($E=16.7$ kv)		($E=17.1$ kv)	
105	0.40	32	1.7	39	1.4
	0.81	57	1.7	63	1.4
	1.32	72	1.8	80	1.3
	2.28	87	1.6	92	1.1
		($E=16.5$ kv)		($E=17.0$ kv)	
122	0.45	43	1.4	46	1.3
	0.83	63	1.5	66	1.3
	1.29	81	1.3	82	1.2
	1.79	88	1.2	91	1.0
	2.28	92	1.1	97	0.5
		($E=18.9$ kv)		($E=19.4$ kv)	
139	0.45	45	1.3	47	1.2
	0.83	62	1.5	65	1.4
	1.29	78	1.4	81	1.3
	1.79	92	0.9	94	0.7
	2.28	95	0.8	98	0.4

found by the various investigators previously mentioned. We will describe later a method of obtaining overvoltages of many times the sparking potential and hence very rapid breakdowns.

In an attempt to analyze the results found by the first experimental method on the basis of Toepler's law we were unable to find an exact solution of the problem. However, it was found possible to obtain an approximation to the solution by a rather long and tedious method that we do not believe worth while to describe here. It will suffice to show a set of data for air at atmospheric pressure in Table III. It will be observed that the agreement is good except for short wire lengths. This divergence, in part at least, perhaps results from the errors introduced by the time lag in the measuring gap G_2. That such errors should appear here and not (appreciably) in the second or

TABLE III. *Comparison between observed values of the maximum potential at G_2 (First Method) for air at atmospheric pressure and those calculated using Toepler's law.*

Wire length in meters	Maximum potential at the open end G_2	
	Observed	Calculated using $K\delta/E=2.0\times10^{-9}$
12	196	195
10	192	192
8	185	189
6	175	183
4	160	173

closed end method is to be expected in consequence of the difference in the multiple reflections for the two cases. The fact that the agreement between calculated and observed values is excellent for long wire lengths bears this out.

Discussion

The principal possible sources of experimental error in both of the two above methods lie in the distortion of the wave as it travels along the wires and in the assumption that the sparking width of G_2 when intensely irradiated measures the maximum applied potential. Fortunately the first of these sources of error can be calculated with good approximation from well-established theory and the corrections applied when large enough to influence the results. The second difficulty, however, gives more serious trouble, but it is nevertheless possible, to investigate this source of error experimentally. In a separate set of experiments in which we measured the time lag of intensely irradiated spark gaps we found the time lag too short to affect appreciably our present results. In the second method it was possible to test directly the effect of time lag. If the capacity C_1 is increased the number of applications of the surge potential that would produce breakdown of G_2 is increased (where, for example, the first surge potential exceeded the steady breakdown value of G_2 by only a few percent). This increased number of applications of the potential should decrease the error due to time lag. Our results showed no variation with increase of C_1 above the value used in the experiment. Further, the rather rapid increase in the rate of fall of potential observed with increasing pressure indicates that the time lag is not a significant factor.

It may seem at first sight remarkable that it is possible to represent the electrical discharge phenomena in its initial stages by a relation as simple as Toepler's resistance law. More careful consideration shows, however, that the relation may be established logically as a result of the simplification introduced by the very short times involved. The discharge current arises from ionizing collisions in which both electrons and positive ions are produced. Due to the great difference in mobility between these, the current in the initial stages of the discharge arises almost entirely from the movement of the electrons. In the short times involved there is comparatively little recombination and the positive ions move only a small distance from where they are generated. A state of ionization then exists in the discharge of an intensity proportional to the quantity of charge which has passed. Hence, to arrive at Toepler's relation it is only necessary to assume that the conductivity of the discharge is proportional to the ionization. The increase in the rate of fall of potential with increase of pressure (over the range studied) is also accounted for by the fact that the sparking potential as well as the number of molecules between the electrodes is increased. Consequently the rate of ionization should increase which, in turn, should lower k in Eq. (2). The marked lowering in every case of $k\delta/E$ for large values of ΔV in Tables I and II, indicates that the amount of ionization ceases to be directly proportional to the charge that had previously passed as the discharge approaches the arc stage.

Discharges at High Field Strengths

We shall now return to the question of obtaining a very rapid fall of potential after once the discharge is initiated. Previously, we[13] have mentioned the possibility of applying as much as 5×10^5 volts per cm for over 10^{-6} sec without electrical breakdown in air, hydrogen and nitrogen at atmospheric pressure. The gas was carefully dried and filtered and the ions kept continually removed by auxiliary fields. Fig. 7 shows a schematic diagram of such an apparatus. The gap G_3 is enclosed in either a metal chamber, or better, a large glass vessel with a cage of fine mesh brass wire screening most of the glass parts as shown by the dotted circle in the figure. The electrodes of G_3 were hard brass balls but when it is not desired to trip the spark with ultraviolet light steel balls can be used. A quartz window is so placed that G_3 can be irradiated as well as its light observed. The metal plates PP are spaced many times the distance between the electrodes in order to reduce capacity effects.

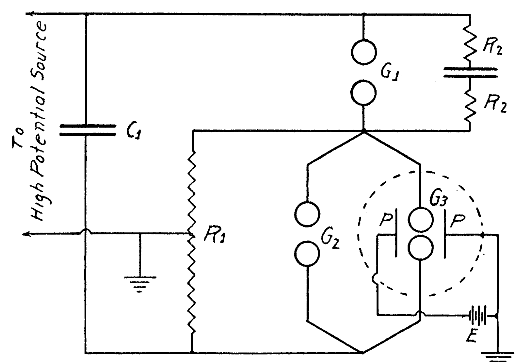

Fig. 7. Schematic diagram of the apparatus used for obtaining discharges at very high field strengths.

[13] Beams and Street, Phys. Rev. 35, 658 (1930).

They are connected to a 200 volt vattery E and used to sweep the ions out of G_3.

The potential is slowly applied to the condensers C_1 and C_2 until G_1 breaks down. When this occurs G_2 and G_3 have approximately equal applied potentials until C_1 discharges. This time of discharge can be regulated by the size of the capacity C_1 and the resistance R_1. If ordinary unfiltered air is surrounding G_3 and all auxiliary ion sweeping removed, G_2 and G_3 have approximately the same sparking width. However, when the gas surrounding G_3 is carefully dried and filtered and a potential applied across PP to remove the ions from G_3 it was found that it was necessary to increase G_1 and G_2 to over 16 times the width of G_3 before G_3 would break down in less than 10^{-6} sec. If ultraviolet light was allowed to fall upon the cathode of G_3 it then sparked at approximately the same width as G_2 regardless of what was done to the gas. As the applied potential necessary to produce breakdown in 10^{-6} sec in G_3, when carefully swept of ions, was exceeded, even by a small amount the time between the application of the voltage and the breakdown was very much decreased. The maximum potential that could be applied across G_3 without breakdown in less than 10^{-6} sec depended upon the treatment of the surface of the electrodes. For example, a discharge in hydrogen at low pressure between the electrodes increased the maximum previous breakdown potential by almost 30 percent. Further, roughness of the electrodes, especially small microscopic projections such as develop from constant sparking, reduces the potential. If the gas is carefully removed from G_3 so that the pressure is less than 10^{-6} mm of mercury and the surge potentials applied as before, the potential necessary to break G_3 down is the same order of magnitude as with the dried filtered gas at atmospheric pressure, carefully swept free of ions.

The explanation of the fact that large fields can be impressed across a gap carefully swept of ions without breakdown seems to be that it is necessary to have an ion in the field to initiate the discharge. The drying and filtering of the air probably permit a more efficient removal of the ions by the auxiliary fields. The dependence of the maximum potential without breakdown in 10^{-6} sec, above described, upon the surface conditions of the cathode suggests that the discharge in ion-free gas is started by electrons pulled out of the cathode by the intense electric fields. When these electrons from the cathode enter the gas they become very efficient in initiating the discharge. Our measurements though only qualitative show that the rate of fall of potential is much faster than in a static breakdown. It seems that such discharges may possibly be utilized in experiments where it is necessary to produce a steep electrical wave or to give information concerning the extraction of electrons from metals by electric fields although the metal is surrounded by gases at various pressures.